

Household equivalence scales and household taxation

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Abstract

Strictly equal utility is seldom used as a compensation principle in real-world policy applications. We specify less radical norms for deriving income tax rates and find that “equivalence” defined as equal proportional sacrifice best fits the observed income tax structure in the Netherlands. We use the Leyden welfare function of income as our measure of utility.

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1 Introduction

Equivalence scales, specifying the utility weight of characteristics of individuals or households, are applied in cases where compensation for these characteristics is a policy goal. For example, equivalence scales for adults and children, differentiated by age, can be used to derive income tax rates or the level of child allowances. Straightforward application of equivalence scales implicitly assumes that perfectly equal welfare irrespective of household composition is a desirable goal. While this indeed has been applied to derive child allowance levels (see Van Praag and Plug, 1993), such a strict egalitarian goal is not commonly applied to the determination of (income) tax rates. In fact, less far reaching criteria are applied, such as equal proportional or equal marginal sacrifice. Indeed, there is probably not a simple example of a policy of full compensation to implement perfectly equal welfare: policy makers seldom apply truly radical rules. But that poses the question which rule does get application, and this is what we set out to discover in this paper. We specify four different equity principles, derive the implied income tax structure, and then assess which principle scores best in explaining the observed income tax rate structure. The four principle are: equal utility, equal marginal utility, equal proportional sacrifice and equal

absolute sacrifice. These four principles have a long history in the theory of optimal taxation.¹ The utility function we use is the Leyden Welfare Function of Income, WFI. This is a direct measure of individual welfare that has extensive empirical support for many countries and conditions. We proceed as follows.

The first section introduces the Leyden welfare function. The second section discusses the four alternative tax recipes and calculates the resulting equivalence scales. The third section discusses the dataset. The fourth section reports on the results. Firstly, different types of equivalence scales are evaluated. We find that equivalence scales in the traditional sense are the most sensitive for household changes. Secondly, the structure of the Dutch income tax system is evaluated in terms of the calculated equivalence scales and equivalent incomes. We find that the Dutch regime of income taxes mirrors a tax regime that is based on the concept of equal relative sacrifice. The final section concludes.

2 Leyden welfare

In this section, we apply the Leyden welfare function of income (Van Praag, 1968) which is identified by means of subjective information. People answer the Income Evaluation Question (Van Praag, 1971) which runs as follows

Which monthly household after tax income would you in your circumstances consider to be very bad? Bad? Insufficient? Sufficient? Good? Very good?

The answers of the IEQ are denoted as c_1, c_2, c_3, c_4, c_5 and c_6 . If we accept that the answers linked to the verbal qualifiers “very bad, bad, insufficient, sufficient, good” and “very good” are evaluations of welfare derived from these various income levels, the IEQ gives us six points on an individual welfare function (Van Praag, 1991). The Equal Interval Assumption (Van Praag, 1971) translates these verbal qualifications on a numerical scale

$$U(c_z) = \frac{2z - 1}{12} \tag{2.1}$$

where z runs from 1 to 6. This follows if the respondent uses the offered labels in such a way that they convey the maximal information on their welfare function of income (Van Praag, 1968). The fact that the labels are placed equidistantly is examined in Buyze (1982) and Van Praag (1991). Both test the Equal Interval Assumption and do not reject it.

If the welfare function $U(y_a)$ is approximated by a lognormal distribution function, see Van Praag (1968) and Van Herwaarden and Kapteyn (1981) for supporting arguments, utility $U(y_a)$ can be written as $\Lambda(y_a; \mu, \sigma)$.² The welfare parameters μ and σ are estimated as

$$\mu = \frac{1}{6} \sum_{i=1}^6 \ln c_i \tag{2.2}$$

and

$$\sigma^2 = \frac{1}{5} \sum_{i=1}^6 (\ln c_i - \mu)^2 \quad (2.3)$$

The two welfare parameters μ and σ may vary over households. We apply the traditional explanation for the differences in μ and write

$$\mu = \beta_0 + \beta_1 \ln fs + \beta_2 \ln y_a \quad (2.4)$$

where

y_a household after tax income
 fs number of persons in the household

The welfare parameter μ is increasing in the argument fs which relates to the fact that children within the household create costs and therefore influence welfare experienced. That is, a family of six will need a higher income to obtain a certain welfare level than a family of four, other things being equal. The parameter is also increasing in the argument y_a . This phenomenon is better known as the preference drift (see Van Praag, 1971) and reflects the way people adapt their income judgment to changes in their after tax income. A positive preference drift says that part of additional income is not contributing to higher welfare since aspirations increase as well. We assume conform to most analyses of the Leyden type that σ is taken to be randomly varying over households (see Hagenaars, 1986).

3 Four taxation regimes

The structure and graduation of taxes has always been a prime topic of economics where desired or fair income distributions are used as guidelines. Some ideas underlying the four desired income distributions will be discussed.

In the early literature a fair or optimal distribution of income (taxes) was often defined in terms of sacrifices to pay. For example, John Stuart Mill viewed the equality of taxation in terms of equality of sacrifice, (see Mill, 1965). There was difference of opinion on which sacrifice should be equal. In this paper we choose for both the concepts absolute and relative sacrifice as a tax base. The other two tax principles are based on a radical egalitarian norm and the maximization of a Benthamite social welfare function. For a more detailed description of the different taxation principles we refer to Plug et al (1999). Note that all these tax principles can be defined in terms of welfare but have different implications. For each tax principle we determine the equivalent incomes and equivalence scales.

Equal utility

For household of type 1, fs_1 , the enjoyment of equal utility derived from income after the deduction of income tax y_{a1} requires

$$U(y_{a1}, fs_1) = \alpha_1 \quad (3.1)$$

where α_1 represents a welfare level. In terms of Leyden welfare, equivalent income reads as

$$\ln y_{a1}(\alpha_1, fs_1) = \frac{\beta_0 + \beta_1 \ln fs_1 + \sigma \Phi^{-1}(\alpha_1)}{1 - \beta_2} \quad (3.2)$$

where Φ is the standard normal cumulative distribution function.

Equal marginal utility

The same household will enjoy equal marginal utility derived from after tax household income

$$\frac{\partial U(y_{a1}, fs_1)}{\partial y_a} = U'(y_{a1}, fs_1) = \alpha_2 \quad (3.3)$$

In terms of Leyden welfare, the marginal utility of income is defined

$$\frac{\partial \Lambda(y_a; \mu, \sigma)}{\partial y_a} = \frac{1}{\sigma y_a} \phi \left(\frac{\ln y_a - \mu}{\sigma} \right) \quad (3.4)$$

where ϕ is the standard normal distribution function. For household of type 1 we define

$$\begin{aligned} \theta_0^1 &= (1 - \beta_2)^2 \\ \theta_1^1 &= 2\sigma^2 - 2(1 - \beta_2)(\beta_0 + \beta_1 \ln fs_1) \\ \theta_2^1 &= (\beta_0 + \beta_1 \ln fs_1)^2 + c \end{aligned} \quad (3.5)$$

where equivalent income reads as

$$\ln y_{a1}(\alpha_2, fs_1) = \left[\frac{-\theta_1^1 + \sqrt{\theta_1^1 - 4\theta_0^1\theta_2^1}}{2\theta_0^1} \right] \quad (3.6)$$

The constant c depends on

$$c = 2\sigma^2 \ln \alpha_2 \sigma \sqrt{2\pi} \quad (3.7)$$

Equal utility ratios (equal proportional sacrifice)

Proportional sacrifice of utility, to the proportion α_3 , requires

$$(1 - \alpha_3)U(y_b) = U(y_a) \quad (3.8)$$

This time equivalent income for household fs_1 reads as

$$\ln y_{a1}(\alpha_3, fs_1) = \left[\frac{\sigma \Phi^{-1}((1 - \alpha_3)U(y_{b1})) + \beta_0 + \beta_1 \ln fs_1}{1 - \beta_2} \right] \quad (3.9)$$

Equal utility differences (equal absolute sacrifice)

Equal sacrifice of absolute utility requires

$$U(y_b) - U(y_a) = \alpha_4 \quad (3.10)$$

Under the tax regime of equal proportional sacrifice we find for household fs_1

$$\ln y_{a1}(\alpha_4, fs_1) = \left[\frac{\sigma \Phi^{-1}(U(y_{b1}) - \alpha_4) + \beta_0 + \beta_1 \ln fs_1}{1 - \beta_2} \right] \quad (3.11)$$

Equivalence scales

For two different households fs_1 and fs_2 the household equivalence scale is defined simply as the quotient between the resulting amounts of equivalent income

$$m(\alpha_i) = \frac{y_a(fs_2, \alpha_i)}{y_a(fs_1, \alpha_i)} \quad (3.12)$$

where i runs from 1 to 4 for the different criteria. Hence, our “equivalence” concept covers the traditional equivalence as equal utility (i equals 1) and the less restrictive equivalence for i equal to 2,3,4.

4 Data description

In 1952 one fourth of the sixth-grade pupils (then about 12 years old) in Holland’s southern province of Noord-Brabant were sampled. In 1983, the same individuals were contacted to collect data on education, labour market status, earnings and so on (some 4700 out of 5800 original observations). As the 1983 edition of the Brabant-data turned out to be a very rich data-set, the 1983-exercise was repeated in 1993. In the latest edition the questionnaire included also questions concerning subjective welfare measurement, one of these being the Income Evaluation Question (IEQ).

In this paper, we restrict ourselves basically to this question for our analysis. The number of Brabant people who responded to the 1993 questionnaire is 2099. From these households a number of observations had to be removed from analysis due to (partial) nonresponse. Due to missings with respect to household current net income y_a we had to remove 437 observations. From the remaining IEQ answers we were able to identify 844 welfare functions. Finally, we corrected for extreme IEQ-response behavior by assuming that “normal” response behavior has to satisfy

$$0.025 \leq N \left(\frac{\ln y_a - \mu}{\sigma} \right) \leq 0.975$$

In words normal respondents evaluate their own current income y_a between the values 0.025 and 0.975 on a $[0,1]$ -scale. The final sample under analysis contains 775 observations.

Table 1: LEYDEN WELFARE FUNCTION (μ)

intercept	3.544	<i>0.148*</i>
ln household size	0.102	<i>0.020*</i>
ln household income	0.522	<i>0.018*</i>
R^2	0.560	
N	775	

Standard errors in italics;

** implies significance at 1% level.

5 The results

Table 1 presents the μ estimates and reports the usual income and family size effects, see Van Praag and Kapteyn (1973). These estimates identify the Leyden welfare function.

Because of unknown α 's we are unable to determine the equivalence scales under the four different tax regimes. Prior to the estimation of the different equivalence scales, we have to consider our present Dutch tax structure. In order to identify the four different tax structures we calculate each of the four α 's so that the actual tax revenue R is equal to the observed total tax revenue. That is, the total sum of observed after tax income y_a equals the total sum of the equivalent incomes

$$\sum_{k=1}^N y_{ka} = \sum_{k=1}^N y_{ka}(\alpha_i) \text{ where } i \text{ runs from 1 to 4} \quad (5.1)$$

The four α 's represent the income taxes, levied according to equal utility, equal marginal utility, equal absolute sacrifice and equal relative sacrifice. We found 0.794, 0.142 (divided by 1000), 0.112 and 0.101 respectively.

Table 2: EQUIVALENCE SCALES

y_b equals Dfl 3000, number of children:	one	two	three	four
equal absolute utility	1.090*	1.159*	1.216*	1.264*
equal marginal utility	1.046*	1.080*	1.106*	1.128*
equal proportional sacrifice	1.015	1.025	1.033	1.038
equal absolute sacrifice	1.007	1.011	1.014	1.015
y_b equals Dfl 4000, number of children:	one	two	three	four
equal absolute utility	1.090	1.159*	1.216*	1.264*
equal marginal utility	1.046*	1.080*	1.106*	1.128*
equal proportional sacrifice	1.023	1.038	1.049	1.057
equal absolute sacrifice	1.016	1.026	1.032	1.037
y_b equals Dfl 5000, number of children:	one	two	three	four
equal absolute utility	1.090	1.159	1.216	1.264
equal marginal utility	1.046	1.080	1.106	1.128
equal proportional sacrifice	1.031	1.051	1.066	1.078
equal absolute sacrifice	1.024	1.040	1.051	1.059
y_b equals Dfl 6000, number of children:	one	two	three	four
equal absolute utility	1.090	1.159	1.216	1.264
equal marginal utility	1.046	1.080	1.106	1.128
equal proportional sacrifice	1.039	1.065	1.084	1.099
equal absolute sacrifice	1.032	1.054	1.069	1.081
y_b equals Dfl 7000, number of children:	one	two	three	four
equal absolute utility	1.090	1.159	1.216	1.264
equal marginal utility	1.046	1.080	1.106	1.128
equal proportional sacrifice	1.046	1.078	1.102	1.120
equal absolute sacrifice	1.040	1.067	1.087	1.102
y_b equals Dfl 8000, number of children:	one	two	three	four
equal absolute utility	1.090	1.159	1.216	1.264
equal marginal utility	1.046	1.080	1.106	1.128
equal proportional sacrifice	1.053	1.090	1.118	1.140
equal absolute sacrifice	1.048	1.080	1.104	1.123

The * points to a negative income tax.

Combining the α 's with the estimates of Table 1 the scales under the four different tax regimes can be identified. The equivalence scales are tabulated in Table 2 for different combinations of monthly gross income and number of children. We see that the effect of children is immediately clear under the equal utility and equal marginal utility rule. As was to be anticipated both do not depend on gross income. In fact, the equivalence scales based on equal utility are uniquely defined as they do not depend on after tax income either. Lewbel (1989) and Blackorby and Donaldson (1991) call this the independent-of-base property of equivalence scales. This cannot be said of equivalence scales based on the equal marginal utility principle. These scales are constant for different gross incomes because they are evaluated for only one marginal welfare level α_2 . Another marginal welfare value would yield different scales. Finally, we see that the equivalence scales based on equivalency in welfare are steeper.

For the other two tax cases, the equivalence scales defined in terms of equal absolute and relative sacrifice show only moderate differences for low gross income levels. For higher incomes, the effect of children becomes more marked. In fact at a gross monthly income of 7000 Dutch guilders, the equivalence scales based on equivalency in marginal utility and proportional sacrifice are almost the same. Resuming, we find that all scales are increasing in the number of children where the most extreme tax criteria (equal marginal and absolute welfare) show the strongest effects.

The Dutch tax system

So far, we have calculated the after-tax income belonging to the four different tax principles (differentiated for household size) and derived the equivalence scales using tax recipes as a measure for equivalency. In theory, these scales show how the four taxes should be targeted at households of different size and structure. Consequently, the question rises how the real taxes perform compared to their theoretical counterparts?

Before we answer, we have to consider our formulas once more. By means of the calculated α 's we were able to determine the four different after tax incomes. To find out which principle predicts best, we simply regress observed after-tax income on predicted after-tax income

$$\ln y_a = \gamma_0 + \gamma_1 \ln y_a(\alpha_i) + \varepsilon \quad (5.2)$$

Equality between practice and theory can be found if the intercept and slope would equal zero and one, respectively. Now, the similarity to the actual tax function can be judged using the estimates reported in Table 3.

We highlight two interesting points. Firstly, we find that both sacrifice recipes (stemming from the early literature) outperform the other two alternatives. That is, they produce a better fit with the actual taxes. Equal utility and equal marginal utility yield correlation coefficients of 0.53, the sacrifice rules yield 0.87. Also in Keller and Hartog (1977) a strong relation is found between

Table 3: ESTIMATING FOUR TAX RECIPES (N=775)

	α	γ_0		γ_1		R ²
equal absolute utility	0.794	-3.754	<i>0.393</i>	1.393	<i>0.047</i>	0.526
equal marginal utility	0.142*	-3.750	<i>0.407</i>	1.445	<i>0.049</i>	0.526
equal proportional sacrifice	0.112	-2.383	<i>0.150</i>	1.283	<i>0.018</i>	0.865
equal absolute sacrifice	0.101	-1.296	<i>0.132</i>	1.153	<i>0.016</i>	0.868

Standard errors are in italics. The α^* is divided by 1000.

the actual taxes and a tax regime of proportional sacrifice.³ Hence, we believe that the principle of equal proportional sacrifice can be seen as a rough guideline underlying the Dutch income tax structure.

Secondly, it is worth noticing that all regressions have a significant negative intercept and a slope significantly above one. This means that all four tax regimes predict a more progressive relation between taxes and income. The richards should be taxed more than they are now and those who are poor should pay lesser or no taxes at all. As a consequence, there is one income level where the actual after tax income exactly coincides with the calculated income level, the cross-over income level. These four monthly after tax income levels resulting from the equal utility, equal marginal utility, equal absolute sacrifice and equal relative sacrifice tax regimes equal 14.075, 4.569, 4.539 and 4.772 Dutch guilders.

6 Concluding remarks

If there is one thing we could learn from the studies of the Leyden type it is that households of different size evaluate the same amount of income differently. Consequently, the same amount of taxes levied over households of different size will be evaluated differently. In practice, we find that most Western countries have child support transfer schemes depending on the number of children and, simultaneously, operate income tax systems that are not household size conditioned.

In this paper we soften the concept of equivalence scales and show how these scales can be utilized as a scientific tool for the evaluation of tax structures. More specifically, we consider four different tax recipes and apply these principles of taxation on the concept of equivalence scales. The alternative equivalence scales are defined so that all households face equal welfare, equal marginal welfare, equal proportional sacrifice and equal absolute sacrifice after the taxes are paid. Hence, “equivalent taxes” are equally evaluated by households of different size in terms of the applied tax regime. Two conclusions emerge.

- The evaluations of income taxes do depend on household characteristics like household size. We find the largest impact of household size on the taxes paid if taxes are levied according to the principle of equal welfare.

This is not surprising as an extreme tax reform (equal welfare) probably yields the strongest effects. The scales based on absolute and relative sacrifice tax principles show for low-income households only moderate effects but are increasing in gross income.

- Although the Dutch tax structure is only partially household size conditioned, we compared our results with the Dutch income tax system. We conclude that all the proposed tax regimes predict a more progressive relation between taxes and income. The richards should be taxed more than they are now and those who are poor should pay less or no taxes at all. Furthermore, we find that despite the discrepancy between the Dutch income tax structure and the proposed alternative tax regimes, the actual taxes show a strong resemblance with taxes paid under a tax regime of equal proportional sacrifice. That is, the Dutch tax structure mirrors a flat tax structure in terms of experienced welfare.

Obviously, this is a result for the Netherlands. In other systems these results may be different.

Notes

¹ Note that we do not link up to the theory of optimal taxation as derived from the balance between equity and efficiency, as founded by Mirrlees (1971). Essentially, we are concerned with equity only. We know of no testable implications of that theory for the observed income tax rates.

² This theory, which is quite different from the existing micro-economic utility theory, was recently under attack by Seidl (1994). A reaction was given by Van Praag and Kapteyn (1994).

³ Cohen Stuart (1889) showed that income tax progression under the rule of equal proportional sacrifice requires that relative marginal utility of income should fall with rising income. Keller and Hartog (1977) postulate a welfare function with constant income elasticity of relative marginal utility. Applying the principle of equal proportional sacrifice, the income tax function could be derived and the parameters estimated from the actual tax function in any given year. Estimation for each of the eleven tax rate regimes in the period 1948-1976 produced an excellent fit, with R^2 above 0.96 in 9 years (it surpassed 0.93 in the other two years).

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Appendix

If we consider the four tax recipes we have to exploit information on both welfare derived from income after taxes and welfare derived from income before taxes. The first utility concept $U(y_a)$ is captured by Van Praag’s welfare concept completely. Hence

$$U(y_a) = \Lambda(y_a; \mu_a, \sigma)$$

where the welfare parameter μ_a is defined as in equation (2.4). The second utility concept $U(y_b)$ is a different story. Because people use income after tax deduction y_a as their anchor point when answering the IEQ, we might question whether the Leyden welfare function is applicable for gross income y_b as well. In this paper we will ignore this difficulty and assume that the Leyden welfare function can be applied for gross income as well. We shape the utility function $U(y_b)$ conform the Leyden tradition and write

$$U(y_b) = \Lambda(y_b; \mu_b, \sigma)$$

where μ_b reads as

$$\mu_b = \beta_0 + \beta_1 \ln fs + \beta_2 \ln y_b$$

The β ’s are the estimates taken from equation (2.4). Note that both utility types imply that in a world without taxation income is equally evaluated.