

Firm-Specific Human Capital as a Shared Investment: Comment

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Employment relationships typically involve the division of surplus. Surplus can be the result of a good match, a monopoly rent, or a quasi-rent that arises because of a specific investment. The division of this surplus is of economic interest as it is a determinant of turnover, investment, and wages. Gary S. Becker (1962) argued, in the context of specific human-capital investments, that the incumbent employee and firm will share the surplus. This notion was formalized in a prototypical model of surplus sharing as first proposed by Masanori Hashimoto (1981). The model has been studied extensively, among others by H. Lorne Carmichael (1983), Robert E. Hall and Edward P. Lazear (1984), and Donald O. Parsons (1986), while Elizabeth Becker and Cotton M. Lindsay (1994) provide an empirical application.

The key feature of the model is the existence of transaction costs. Both the employee and the firm have (*ex ante* uncertain) private information on which they cannot write a contingent contract. This makes that they write a nonrenegotiable contract that specifies a fixed wage. After this, the firm learns the value of the employee's marginal product whereas the employee learns the value of his outside option. Both parties then decide unilaterally whether to separate or not and inefficient separations may occur. The wage is set in such a way as to maximize the expected total surplus.

The present analysis will consider the role of uncertainty in this model. This has not been done before in a rigorous way. Hashimoto considered only degenerate cases. His analysis suggests that the wage will be low when the

uncertainty of the market conditions is small, and high when the uncertainty of the conditions inside the firm is small. Parsons (1986) makes a claim that this is actually the case without deriving the result.

This paper shows that the comparative statics are ambiguous and may well be the opposite of those suggested by Hashimoto and claimed by Parsons. It also provides the intuition that is behind this result, namely that uncertainty not only influences turnover but also the option value of the match and its opportunity cost. Section I, briefly summarizes Hashimoto's model and shows that without further assumptions the comparative statics are ambiguous. Section II derives an explicit solution of the wage. Section III briefly considers alternative wage-setting schemes and Section IV concludes.

I. The Model

The exposition here considers the division of an *ex ante* uncertain surplus and is a simplified version of Hashimoto's (1981) formulation of the model.¹ There are two parties, an employee and a firm, and two periods. In period 1 some specific capital has developed which represents a surplus with value $m + \eta$ if trade takes place in period 2 ($m > 0$). At the start of the second period the firm receives private information concerning the state of product demand denoted by η , whereas the employee receives private information about the value of his market alternative which is denoted by ε . Information is bilaterally asymmetric and as a consequence the employee and the firm cannot contract on the

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¹ In Hashimoto (1981) the worker and the firm also choose the level of investment. The model is closed by imposing a zero-profit condition which implies that the parties' shares in the costs of training equal their respective shares in the expected benefits of training. The sharing decision is, however, independent of the investment level. The simplifying assumption made here that there is a surplus to be divided does therefore not affect the results.

values of η and ε . Instead they write down in advance the terms of employment in the form of a fixed wage for period 2 (w). Renegotiation is not possible. After having learned η , the firm has then to decide whether or not to dismiss the employee. The firm will lay off the employee if the surplus is less than the wage that the employee will receive in case of employment. This gives:

$$\eta \leq w - m.$$

Similarly, after learning ε the employee has to decide whether to quit or to stay with the current firm. The employee quits if the value of his market alternative exceeds the wage:

$$\varepsilon > w.$$

Dismissals and quits are irreversible. Note that efficient separation takes place if

$$m + \eta \leq \varepsilon$$

which means that the parties will separate if the size of the surplus $m + \eta$ is less than the gain of a separation ε . Inefficient separations may occur since $\eta \leq w - m$ or $\varepsilon > w$ can hold while $m + \eta > \varepsilon$.

Although the employee does not know η and the firm does not know ε , their distributions are common knowledge, and $E[\varepsilon] = E[\eta] = 0$. It is assumed that η and ε are independent. The probability of a quit can now be written as $Q = \Pr(\varepsilon > w)$ and the probability of a layoff as $L = \Pr(\eta \leq w - m)$.

The employee and the firm share the surplus in such a way that they maximize the expected total *ex post* surplus. The surplus is shared through the wage. It is assumed that the parties are risk neutral, that there is no discounting, and that capital and labor markets are perfect. The employee's expected surplus is given by

$$G_E = (1 - L)(1 - Q)(w - E[\varepsilon | \varepsilon \leq w])$$

which equals the probability that neither party

separates, times the wage minus the opportunity cost of the match. The expected return to the employee in case of a layoff equals zero.²

The firm's expected surplus is similarly given by

$$G_F = (1 - L)(1 - Q) \\ \times (m + E[\eta | \eta > w - m] - w).$$

This is the probability that neither party separates, times the certain value of the match plus the option value of the match minus the wage. The wage is chosen to maximize the total expected *ex post* surplus G which equals $G_E + G_F$.

This gives the following objective function:

$$(1) \quad \max_w G = (1 - L)(1 - Q) \\ \times (m + E[\eta | \eta > w - m] \\ - E[\varepsilon | \varepsilon \leq w]).$$

The expected total surplus equals the probability that neither party separates times the certain value of the match plus the option value of the match minus the opportunity cost of the match.

Letting lowercase subscripts denote partial derivatives, the first-order condition of the optimization problem is (after some rewriting):

$$(2) \quad G_w = -L_w(1 - Q)(w - E[\varepsilon | \varepsilon \leq w]) \\ - Q_w(1 - L) \\ \times (m + E[\eta | \eta > w - m] - w) \\ = 0.$$

Inspection of (2) shows that w is chosen such as to balance the losses from a suboptimal dismissal and a suboptimal quit.

² There are in fact two possibilities: (i) the firm lays off and the employee quits, and (ii) the firm lays off and the employee does not quit. The associated expected surplus equals $QLE[\varepsilon | \varepsilon > w] + (1 - Q)LE[\varepsilon | \varepsilon \leq w] = LE[\varepsilon] = 0$.

A. The Employee's Sharing Ratio³

Although the analysis up to here considers the wage, one might argue that it is more interesting to analyze the share that the employee obtains. Hashimoto (1981) defined the employee's sharing ratio as the wage divided by the unconditional expected value of the surplus:

$$\alpha = \frac{w^*}{m}.$$

The problem with this definition is that it is not a share. It can take on values greater than one as well as less than zero (the employee pays for employment). The reason for this is that the degree of uncertainty of, say, market demand determines the size of the expected *ex post* surplus G . If one is interested in the share that the employee obtains then it is necessary to correct for this fact. The logical measure would be the employee's share of the expected *ex post* surplus:

$$(3) \quad \alpha_E = \frac{G_E}{G} = \frac{w - E[\varepsilon | \varepsilon \leq w]}{m + E[\eta | \eta > w - m] - E[\varepsilon | \varepsilon \leq w]}.$$

Rearrangement of the first-order condition (2), gives the following equality:

$$\frac{w^* - E[\varepsilon | \varepsilon \leq w^*]}{m + E[\eta | \eta > w^* - m] - E[\varepsilon | \varepsilon \leq w^*]} = \frac{-Q_w/(1 - Q)}{L_w/(1 - L) - Q_w/(1 - Q)}.$$

It must be emphasized that this is a first-order condition and that the left-hand side as well as the right-hand side of the equality are functions of w^* . This expression shows that α_E lies between 0 and 1. It also illustrates that the em-

ployee's sharing ratio depends on the local characteristics of the distributions of η and ε .

B. Comparative Statics

The relevant literature has largely neglected comparative statics in this model. Hashimoto does not derive any explicit comparative statics. Instead he discusses three special cases. The first is when ε and η are both degenerate at zero; in that case the firm will never dismiss the employee and the employee will never quit. Hence, how they share the surplus is immaterial. The second special case is when ε alone is degenerate at zero. In that case the employee will not quit but can be laid off; to minimize the probability of an inefficient layoff the employee will not share the surplus. Similarly, w equals m if η is degenerate at zero (the firm never lays off). In this case the the firm does not share the surplus because the employee might inefficiently quit.

The last two special cases suggest that optimal w increases with the dispersion of ε and decreases with the dispersion η . Although Hashimoto is not explicit about this, Parsons (1986) writes, in his contribution to the *Handbook of Labor Economics*: "it is easily demonstrated that ... as the variance of η increases relative to that of ε , the firm should optimally undertake an increasing share of the investment" (p. 826).⁴

To see whether this is the case one should take the differential of (2). Doing this with respect to the variance of η shows that this does not give a definite answer.

$$(4) \quad \frac{dw^*}{d\sigma_\eta} = \underbrace{((1 - Q)(w^* - E[\varepsilon | \varepsilon \leq w^*])\partial L_w / \partial \sigma_\eta)}_{+} + \underbrace{E[\eta | \eta > w^* - m] - w^*}_{+} \underbrace{\partial L / \partial \sigma_\eta}_{+/-} - \underbrace{Q_w(m + E[\eta | \eta > w^* - m] - w^*)\partial L / \partial \sigma_\eta}_{-} + \underbrace{Q_w(1 - L)\partial E[\eta | \eta > w^* - m] / \partial \sigma_\eta}_{-} / \underbrace{G_{ww}}_{-}.$$

Increasing σ_η has three effects: there is an

³ This subsection benefited much from the insightful suggestions from the anonymous referee.

⁴ Becker and Lindsay (1994) base their empirical test on Parsons' conjecture.

effect on the probability of an inefficient layoff, an effect on the probability of an inefficient quit, and an effect on the loss in case of an inefficient quit. The signs under the braces give the direction of the term involved. Only the effect on the probability of a layoff can take on both a negative and a positive sign depending on the distribution involved, thereby making the sign of (4) indeterminate. This shows that any comparative statics will depend on the distributions of η and ε . The next section gives a simple example for which (4) is always positive and provides the intuition for this seemingly counterintuitive result.

II. An Example

It is possible to derive an explicit solution of the optimal wage after making assumptions about the distributions functions from which ε and η are drawn. More specifically, assume that ε is drawn from a uniform distribution over the interval $[-e, e]$, and that η is drawn from a uniform distribution over the interval $[-t, t]$. We further assume that these distributions are nondegenerate ($e, t > 0$), and that there is a positive probability of separation ($e > m - t$). The model can now be solved and this gives the following result.

PROPOSITION: *Given the assumptions of Section I and the distributional assumptions, the optimal wage w^* is given by the following expression:*

$$(5) \quad w^* = \frac{m + t - e}{2}.$$

PROOF:

Using the distributional assumptions for ε and η it follows that $L = \Pr(\eta < w - m) = (w - m + t)/2t$, $Q = \Pr(\varepsilon > w) = (e - w)/2e$. This in turn implies that $L_w = 1/2t$, and $Q_w = -1/2e$. It is also easily shown that $E[\eta | \eta > w - m] = (t + w - m)/2$, and $E[\varepsilon | \varepsilon \leq w] = (w - e)/2$. Substituting these expressions in (2) gives:

$$-\frac{1}{2t} \left(\frac{e + w}{2e} \right) \left(w - \frac{w - e}{2} \right)$$

$$+ \frac{1}{2e} \left(\frac{t + m - w}{2t} \right) \left(m + \frac{w - m + t}{2} - w \right) = -\frac{1}{2t} \left(\frac{e + w}{2e} \right) \left(\frac{e + w}{2} \right) + \frac{1}{2e} \left(\frac{t + m - w}{2t} \right) \left(\frac{m + t - w}{2} \right) = 0.$$

This simplifies to

$$-(e + w)^2 + (t + m - w)^2 = 0$$

for which (5) is the unique solution.

This result shows that the optimal wage w^* decreases with the uncertainty in the market, and increases with the uncertainty in the firm; $\partial w^*/\partial e < 0$ and $\partial w^*/\partial t > 0$. This finding is the exact opposite of what Hashimoto suggests and Parsons claims. What is the mechanism behind this finding?

The intuition is straightforward: the firm can avoid losses resulting from a bad draw of η by firing the employee. Losses are therefore truncated at zero. Good realizations on the other hand can be captured as long as the employee stays with the firm and grow without bound as t increases ($\partial E[\eta | \eta > w^* - m] / \partial t > 0$). In order to lower the probability that the employee quits, w is increased. For $\partial w^*/\partial e < 0$, the intuition is similar. The employee can capture any good realization of ε by quitting. Bad draws of ε on the other hand are not taken as long as the firm does not fire the employee. To lower the probability of a layoff, w^* will be decreased.⁵

For the specific case considered in this sec-

⁵ For the degenerate cases $t = 0$ ($e = 0$), the results derived by Hashimoto continue to hold. In this case the wage will be set to m (0) because the firm (worker) never lays off (quits). Consequently w is independent of e (t) and therefore the mechanism described in this paragraph is no longer at work. If both $t = 0$ and $e = 0$, the wage will be indetermined and lies in the interval $[0, m]$ because then no inefficient separations occur. The case where $e < m - t$ is conceptually equivalent to the case where $t = 0$ and $e = 0$; by setting a wage between e and $m - t$ no inefficient separations will occur.

tion it turns out that the employee's share of the expected *ex post* surplus $\alpha_E = (w^* + e)/(m + t + e)$. Substitution of the optimal wage w^* from (5) gives $\alpha_E = 1/2$. Hence the expected surplus is maximized when the parties set a wage that splits this expected surplus equally.

The analysis here applies to the case of uniform distributions. It is not possible to derive general results for the signs of $dw^*/d\sigma_\eta$ and $dw^*/d\sigma_\varepsilon$. Numerical analysis using normal distributions shows that both can be either positive and negative. For reasonable values of σ_η and σ_ε , however, the comparative static results are identical to those derived for the uniform case. More importantly, the main mechanism driving the results remains the same. If $|w - m| < \sigma_\eta$ then $dL_w/d\sigma_\eta < 0$ and therefore the sign of $dw^*/d\sigma_\eta$ is always positive. Only if σ_η is relatively small, the option value of the match loses its importance and then it can be the case that the sign of $dw^*/d\sigma_\eta$ is reversed.

III. Alternative Wage-Setting Schemes

With *ex ante* wage setting the employee and the firm set a wage before they observe the realizations of η and ε . One can, however, imagine that the farmer who sets out in the morning to hire farmhands for the day knows quite precisely the value of the labor that he is getting or, in the language of the model, he knows η .⁶ This situation corresponds to the "firm sets wage" contract analyzed by Hall and Lazear (1984), where the firm announces a wage demand after observing η (but not ε) and the worker accepts that wage or does not work for the firm at all. In addition to this "firm sets wage" contract these authors also analyze a "worker sets wage" contract. Here the worker announces a wage demand after observing ε (but not η) and the firm employs the worker at that wage or not at all.

In this section we analyze the role of uncertainty under these two unilateral (*ex post*) wage-setting contracts, maintaining the assumption that η and ε follow uniform distributions. First consider the case where the firm is allowed to

make a take-it-or-leave-it offer after having observed η . The firm's offer will be such that it maximizes the difference between actual productivity and wage times the probability that the employee does not quit.

$$\max_w (1 - Q)(m + \eta - w).$$

This gives the firm's optimum offer

$$w_F^* = \frac{m + \eta - e}{2}.$$

Likewise, if the employee is allowed to make a take-it-or-leave-it offer, he will choose a value such that it maximizes the difference between the wage and the market alternative times the probability that the firm accepts this demand.

$$\max_w (1 - L)(w - \varepsilon).$$

This gives the employee's optimum demand as

$$w_E^* = \frac{m + t + \varepsilon}{2}.$$

Since $\eta \leq t$ and $\varepsilon \geq -e$, it is clear that $w_F^* \leq w^* \leq w_E^*$. That is: the employee's share of the investment in training when the parties maximize their joint gain lies between the shares which result from situations in which one of the parties is allowed to set the wage unilaterally. w^* is closer to w_E^* if t exceeds e , and is closer to w_F^* otherwise. Furthermore, $\partial w_F^*/\partial e < 0$ and $\partial w_E^*/\partial t > 0$, which shows that the main result of this paper holds for *ex post* wage-setting schemes as well.

IV. Conclusion

This paper shows that the employee's wage is high when the conditions inside the firm are uncertain relative to the employee's market alternative, whereas the wage should be small when the employee's market alternative is uncertain relative to the conditions inside the firm. These results are intuitive. Higher un-

⁶ When the productivity of the workers depends, for example, on the weather conditions and the equipment they work with.

certainty of the conditions in the firm increases the option value of the match. The firm can avoid losses resulting from bad conditions inside the firm by firing the employee. Losses are therefore truncated at zero. This means that the returns increase with uncertainty. But good realizations can only be captured as long as the employee stays with the firm. To lower the probability that the employee quits, he is given a higher wage.

Likewise, the employee can capture any good market offer by quitting. Bad market offers on the other hand are not taken as long as the firm does not fire the employee. As a result of more uncertain market alternatives for the employee, the opportunity cost of the match decreases and consequently the surplus increases. To lower the probability that the match dissolves through a layoff, the wage is decreased.

Although the explicit expression for the wage depends on simplifying assumptions regarding the probability distributions of the conditions inside the firm and the conditions in the market, the mechanism that drives our result is more general and does not crucially depend on distributional assumptions or wage-setting institutions.

REFERENCES

- Becker, Elizabeth and Lindsay, Cotton M.** "Sex Differences in Tenure Profiles: Effects of Shared Firm-Specific Investment." *Journal of Labor Economics*, January 1994, 12(1), pp. 98–118.
- Becker, Gary S.** "Investment in Human Capital: A Theoretical Analysis." *Journal of Political Economy*, October 1962, 70, pp. 9–49.
- Carmichael, H. Lorne.** "Firm-Specific Capital and Promotion Ladders." *Bell Journal of Economics*, Spring 1983, 14(1), pp. 251–58.
- Hall, Robert E. and Lazear, Edward P.** "The Excess Sensitivity of Layoffs and Quits to Demand." *Journal of Labor Economics*, April 1984, 2(2), pp. 233–57.
- Hashimoto, Masanori.** "Firm-Specific Human Capital as a Shared Investment." *American Economic Review*, June 1981, 71(3), pp. 475–82.
- Parsons, Donald O.** "The Employment Relationship: Job Attachment, Work Effort, and the Nature of Contracts," in Orley Ashenfelter and Richard Layard, eds., *Handbook of labor economics*, Vol. 2. Amsterdam: North-Holland, 1986, pp. 789–848.